THE APPLICATION OF THE THEORY OF EQUIVALENT CIRCUITS TO THE COMPUTER-AIDED DESIGN OF MICROWAVE CIRCUITS

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Abstract

The field of equivalent circuit generation is examined in relation to microwave circuits. A method useful for microwave circuit design is explained. The method can be regarded as optimization with inequality constraints, but has the advantage of being linear, thus efficient and easy to obtain linear programming routines can be used to do the design by computer.

Introduction and Background

One method of designing circuits is to start with a circuit that meets the design objectives but has some undesirable characteristic (for example being difficult to construct) and to generate from it equivalent circuits which also meet the design objectives but are acceptable.

In 1929 Cauer[1] introduced a method for generating equivalent circuits through linear transformations of the voltages and currents of the circuits. The transformation was such that it transformed the loop-impedance or node-admittance matrices by means of a congruent transformation. For circuits in which ideal transformers are acceptable the Cauer transformation is very useful since it has the property that if the original circuit has a matrix that is symmetric and positive-definite (or semidefinite), the final matrix will have the same characteristics. Since symmetry and positive semi-definiteness is a sufficient condition for a matrix to be realizable with positive elements (R's, L's, C's) and ideal transformers, the final circuit will also be realizable automatically.

In the design of microwave circuits the ideal transformer is not an acceptable element hence it is desirable to be able to insure the realizability of the final matrix without them. It is possible in some cases[2,3] to use transformation matrices that will insure that a condition stronger than positive semi-definiteness, namely hyperdominancy, is satisfied by the transformed matrix, thereby insuring realizability without ideal transformers. Regretfully the cases in which this can be done are quite restricted and of no use in microwave circuit design.

In a series of papers Murray-Lasso and others[4-6] have used a single-sided transformation for the generation of equivalent circuits. Although this transformation is less general than Cauer's and another transformation used by Murray-Lasso , it has the advantage that the transformation coefficients appear linearly in the final matrix and hence linear, instead of non-linear, inequalities have to be solved to impose hyperdominancy to the transformed matrix. In a recent paper a new technique was introduced which allows the structure of the transformed network to be easily controlled. The technique also allows almost equivalent instead of exactly equivalent circuits to be generated This new technique is applicable to the design of microwave circuits and it will be briefly described below.

New Design Method

Let us assume it is desired to design a microwave circuit to match a termination so we are given an input impedance $z(f)$ in both magnitude and phase at a set of discrete frequencies f_1, f_2, . . . f_n. It is desired to synthesize the impedance using a specific structure and elements of a specified nature but whose parameters are as yet unknown. To solve the problem we proceed as follows:

Model the structure with lumped elements (using as many as necessary to achieve a good approximation). Choose the nodes at which the impedance should be realized as nodes 1 and the datum node. For example, assuming the structure is a tapped line, the model might look something like the circuit shown in Fig. 1. Notice that in Fig. 1 all the in uctors and capacitors marked with the same letter and index have equal values. This is important in microwave applications where we try to model distributed devices and desire certain symmetries. In actual fact in Fig. 1 there are only 9 unknowns.

Now write a 13x13 impedance matrix $Z = (z_{ij})$ (Here 13 is the number of nodes of the circuit) with z_{11} equal to the desired $z(f)$ at the frequencies of interest and all the other elements arbitrarily chosen. This matrix is the open-circuit

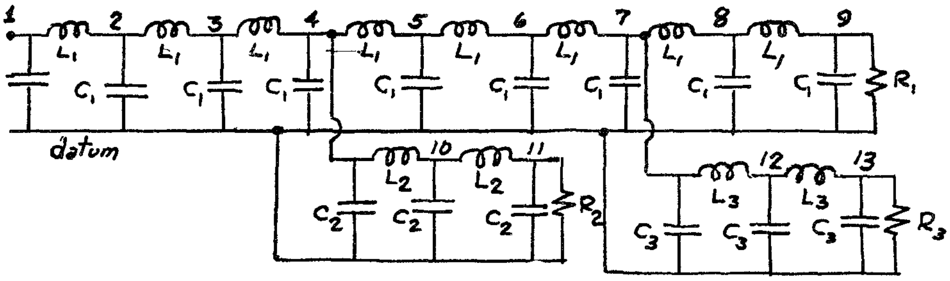

FIGURE 1

impedance matrix of a circuit with ports defined between each node and the datum. If this matrix is now inverted at each frequency of interest, the resulting matrix will be the nodal admittance matrix Y of a network having a driving-point impedance between nodes 1 and the datum equal to the desired z(f). The rest of the circuit will not correspond to the structure of Fig. 1. To change all the driving-point and transfer impedances of the matrix Y so they will ressemble as closely as possible the structure of Fig.1, apply a transformation by postmultiplying Y by a matrix B of the form

$$B = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ b_{21} & b_{22} & b_{23} & \cdots & b_{2,13} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{13,1} & b_{13,2} & b_{13,3} & \cdots & b_{13,13} \end{bmatrix}$$

which will keep z_{11} invariant. We can also impose the structure, and allow some freedom to obtain quasi-equivalent circuits by introducing a complex error matrix E all of whose entries are equal to e. The problem can be stated mathematically as a linear programming problem as follows:

$$\text{minimize} \quad e$$

$$\text{subject to:} \quad YB - \sum_i T_i H_i(f)\, a_i \geq -E$$

$$YB - \sum_i T_i H_i(f)\, a_i \leq E \qquad (1)$$

$$e \geq 0, \quad a_i \geq 0,$$

In (1) the unknowns are the b_{ij} (elements of the matrix B), a_i and e. Since the relations involve complex numbers it will be necessary to separate the real and imaginary parts. The inequalities should be interpreted component by component and real and imaginary parts separately. The matrix E is a 13x13 complex matrix all of whose elements are equal to e. The matrices T_i are topological matrices whose entries are 0, 1, or -1 and indicate where each component is connected. The functions $H_i(f)$ are the frequency dependency of the admittance of each element type. For example, for the inductors $H(f) = 1/2\pi fj$. The real numbers a_i represent the unknown values of the parameters. For the capacitors it is the capacitance, for the inductors it is the elastance and for the resistors it is the conductance.

The problem posed by (1) can be easily solved by computer by means of linear programming routines which are widely available in most computers as library programs.

Discussion

The method given here could be looked upon as optimization with constraints where e is being minimized subject to the fact that the resulting circuit is quasi-equivalent to the original circuit and that the parameters are positive (or within given limits to insure the devices can be easily constructed). The advantage of the method presented here over conventional optimization methods for microwave circuit design is that the one presented is linear, which has theoretical, computational and practical (easy to get as library routine) advantages.

References

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